Spectral bounds obtained by reweighting entries in a row of a nonnegative matrix

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For a square matrix C, the spectral radius $\rho(C)$ is defined as

$$\rho(C) := \max\{\ |\lambda|\ \ |\ \lambda \text{ is an eigenvalue of } C\},$$

where $|\lambda|$ is the magnitude of complex number λ. It is well known that

$$0 \le C \le C' \quad \Rightarrow \quad \rho(C) \le \rho(C'),$$

where C' is another square matrix of the same size. Now assume that C' has the the same row-sum sequence of C, C' has a positive eigenvector $v = (v_1, v_2, \ldots, v_n)^T$ for $\rho(C')$ with the i-th entry the least (i.e. $v_i \le v_j$ for all j), and $C'[-|i)$ is the submatrix of C' obtained by deleting the i-th column. We will show that

$$0 \le C[-|i) \le C'[-|i) \quad \Rightarrow \quad \rho(C) \le \rho(C').$$

Modifying the proof, we also obtain the dual statement that

$$C[-|i) \ge C'[-|i) \ge 0 \quad \Rightarrow \quad \rho(C) \ge \rho(C').$$

Under the assumption that C is irreducible, the necessary and sufficient conditions in terms of C, C', and v for the above two equalities are provided. We provide a way to construct the above C' in which the position i mentioned above is known. When C' has suitable equitable quotient, we obtained a realization of some earlier bounds in the literature [**?**, **?**, **?**, **?**, **?**, **?**] by the spectral radius of matrices of smaller sizes. We will provide new applications. This is a joint work with Chih-wen Weng.

Keywords: spectral radius, nonnegative matrix, row-sum sequence, equitable quotient.

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